IOPscience iopscience.iop.org

Home Search Collections Journals About Contact us My IOPscience

Non-singular cosmology as a result of a spontaneous breakdown of symmetries

This article has been downloaded from IOPscience. Please scroll down to see the full text article.

1981 J. Phys. A: Math. Gen. 14 3039

(http://iopscience.iop.org/0305-4470/14/11/025)

View the table of contents for this issue, or go to the journal homepage for more

Download details:
IP Address: 129.252.86.83
The article was downloaded on 31/05/2010 at 05:39

Please note that terms and conditions apply.

J. Phys. A: Math. Gen. 14 (1981) 3039–3045. Printed in Great Britain

Non-singular cosmology as a result of a spontaneous breakdown of symmetries

S V Orlov
State Committee For Standards, Moscow, USSR

Received 17 April 1980, in final form 13 April 1981

Abstract. Spontaneous breakdown of *C*, *P*, *CP*, *T*, *CPT* and *G* symmetries of the scalar field with a conformal coupling is shown to lead to the absence of an initial singularity in an open-type homogeneous isotropic cosmology. Considering a cosmology with a Λ term, the conclusion about the absence of a singularity is not changed. We treat a self-consistent system of gravitational and scalar fields.

At the present time the problem of a singularity in a cosmology is of great interest owing to many attempts to calculate quantum effects at the early stages of the evolution of the universe (see for instance Stanukovich (1965)). The most interesting effects, which, in principle, may prevent a singularity, are particle creation by non-stationary gravitational fields, filling the universe.

We are going to treat spontaneous breakdown of symmetries in the framework of the self-consistent system of equations of the Einstein gravitational field and a scalar field with a conformal coupling and self-interaction. This system (without self-interaction) was suggested by Bronnikov *et al* (1968) and Callan *et al* (1970). In this work we shall prove that the account of the vacuum properties leads to radical cosmological consequences, i.e. to avoidance of a singularity. Earlier Melnikov and Orlov (1979) have shown that cosmological singularity is avoided if there is *C* invariance spontaneously broken in our universe. Here we shall treat neutral and charged φ-fields, both massive and massless.

Let the Lagrangian be $L=L_g+L_\varphi$, $c=\hbar=1$, where $L_g=(R-2\Lambda)/2k$, Λ is a cosmological term,

$$L_\varphi = g^{\alpha\beta}\nabla_\alpha\varphi^*\nabla_\beta\varphi-(m^2+R/6)\varphi^*\varphi-\tfrac{1}{6}\lambda(\varphi^*\varphi)^2. \tag{1}$$

The sign of the curvature tensor is chosen as in Chernikov and Tagirov (1968). The treatment will be carried out in the metric of an open-type cosmology:

$$\mathrm{d}s^2=a^2(\eta)\left(\mathrm{d}\eta^2-\frac{\mathrm{d}r_1^2}{1+r_1^2}-r_1^2(\mathrm{d}\theta^2+\sin^2\theta\,\mathrm{d}\varphi_1^2)\right) \tag{2}$$

where the preferred cosmological time t and a variable η are connected by $\mathrm{d}t=a\,\mathrm{d}\eta$. The equation of the scalar field obtained from the initial Lagrangian is

$$\Box\varphi+(m^2+R/6)\varphi+\tfrac{1}{3}\lambda\varphi^*\varphi^2=0. \tag{3}$$

The gravitational field is considered to be classical, but φ-field quantised (Bronnikov *et al* 1968, Callan *et al* 1970), so in the right-hand side of the Einstein equation we have

0305-4470/81/113039+07\$01.50 © 1981 The Institute of Physics

the vacuum expectation value of the energy–momentum tensor (Grib and Mostepanenko 1979). The Einstein equations with so-called 'modified' energy–momentum tensor $T^{\nu}_{(s)\mu}$ correspond to our Lagrangian (Chernikov and Tagirov 1968):

$$R^{\nu}_{\mu}-\tfrac{1}{2}\delta^{\nu}_{\mu}R+\delta^{\nu}_{\mu}\Lambda=-k\langle 0|T^{\nu}_{(s)\mu}|0\rangle. \tag{4}$$

If we neglect vacuum fluctuations according to Lee (1974) and take into account the homogeneity of the metric, we may write

$$\langle 0|\varphi^*(\boldsymbol{x},\eta|0\rangle\approx\langle 0|\varphi(\boldsymbol{x},\eta)|0\rangle=\langle 0|\varphi(0,\eta)|0\rangle\equiv(3/\lambda)^{1/2}f(\eta)/a(\eta) \tag{5}$$

where, for simplicity, we put a phase of the vacuum expectation value equal to zero. Using (5), we obtain the scalar field equation in the form (6a) and the $\binom{0}{0}$-component of the Einstein equations when using (2):

$$\ddot{f}+(m^2a^2-1)f+f^3=0, \tag{6a}$$

$$\tfrac{1}{2}\dot{a}^2-\tfrac{1}{2}a^2-\tfrac{2}{3}\Lambda a^4/4=(k/6)a^4\varepsilon_s(\eta), \tag{6b}$$

where the scalar field's energy density (Grib and Mostepanenko 1979) is

$$\varepsilon_s(\eta)=(3f^2/\lambda a^4)(m^2a^2-1+f^2/2+\dot{f}^2/f^2). \tag{7}$$

Suppose that a non-singular cosmology will take place if the universe is filled by the scalar field, i.e. let us search for a solution of (6) up to η^2:

$$f=F(1+x\eta+y^2\eta^2), \tag{8}$$

$$a=a_m(1+\nu\eta^2). \tag{9}$$

Substituting these solutions in (6) and comparing coefficients of the same degrees of η, we obtain

$$x=0, \tag{10}$$

$$-y^2_{\pm}=\tfrac{1}{2}(m^2a^2_m-1)+F^2_{\mp}/2, \tag{11}$$

$$F^2_{\mp}=\tfrac{2}{3}(1-m^2a^2_m)\mp\tfrac{1}{3}[(1-m^2a^2)^2+12m^2a^2_m\nu_{\mp}]^{1/2}. \tag{12}$$

The signs of y^2, ν and F^2 correspond to the signs before the root in (12). From physical considerations,

$$F^2_{\pm}>0,\qquad \nu_{\pm}>0, \tag{13}$$

because we put a phase of vacuum expectation value equal to zero and because of the universe's expansion. The restrictions (13) lead to

$$0<m^2k(1-m^2a^2_m)/3\lambda+[1+\tfrac{2}{3}(\Lambda/m^2)m^2a^2_m]/2<\nu_+, \tag{14a}$$

$$0<\nu_-<m^2k(1-m^2a^2_m)/3\lambda+[1+\tfrac{2}{3}(\Lambda/m^2)m^2a^2_m]/2, \tag{14b}$$

$$\nu_-<(1-m^2a^2_m)^2/m^2a^2_m4. \tag{14c}$$

Then ν is given by

$$\nu=\{[1+\tfrac{2}{3}(\Lambda/m^2)m^2a^2_m]/2-m^2k(1-m^2a^2_m)^2/4\lambda+m^2a^2_m+(\Lambda/m^2)m^4a^4_m/6\}$$
$$\times\{(1-m^2a^2_m)-m^2a^2_mm^2k/\lambda\}^{-1}. \tag{15}$$

Let us denote $m^2a^2_m=r$ and $n=m^2k/\lambda$; then we have the equation for r

$$r^4e_4+r^3e_3+r^2e_2+re_1+e_0=0 \tag{16}$$

where

$$e_4 = 5n^2/4 + n^3 + (\Lambda/m^2)^2(1+2n/3)^2 - (\Lambda/m^2)(7n+8n^2+2n^3), \quad (17a)$$

$$e_3 = (\Lambda/m^2)(4n^3/3+6n^2+32n/3+6) - (3n^3+10n^2+7n), \quad (17b)$$

$$e_2 = 5n^3 + 16.5n^2 + 20n + 9 - (\Lambda/m^2)(7n/3+2n^2), \quad (17c)$$

$$e_1 = -(n^3+8n^2+7n), \qquad e_0 = 5n^2/4. \quad (17d)$$

(16) is the equation for a definition of a scale factor of the initial state a_m. We see from the scalar field equation (6a) that the spontaneous breakdown can exist only at small times, and only then when

$$0 < r < 1. \quad (18)$$

The case with tachyons, i.e. $r < 0$, will not be treated. The energy density (7) takes the form

$$\varepsilon(\eta) = (m^4 3/\lambda)(E_0 + E_2\eta^2), \quad (19a)$$

$$E_0 = (r - 1 + F^2/2)(F^2/r^2), \quad (19b)$$

$$E_2 = [(y^2 - \nu)(r - 1 + 2F^2) + y^4 + 3\nu](F^2/r^2). \quad (19c)$$

We solve the equation (16) using a computer, for the values of n and Λ/m^2

$$\Lambda/m^2 = 0, \pm 1, \quad (20)$$

$$n = (1, 2, \ldots, 10)10^b, \qquad b = -2, -1, 0, +1. \quad (21)$$

Then it is possible that we may obtain two branches of solutions, for example: F_-^2, ν_-, y_+^2 and F_+^2, ν_+, y_-^2 and also E_0, E_2 corresponding to them. Below we adduce the diagrams of them as functions of n and Λ/m^2. These branches are given by two solutions of (16).

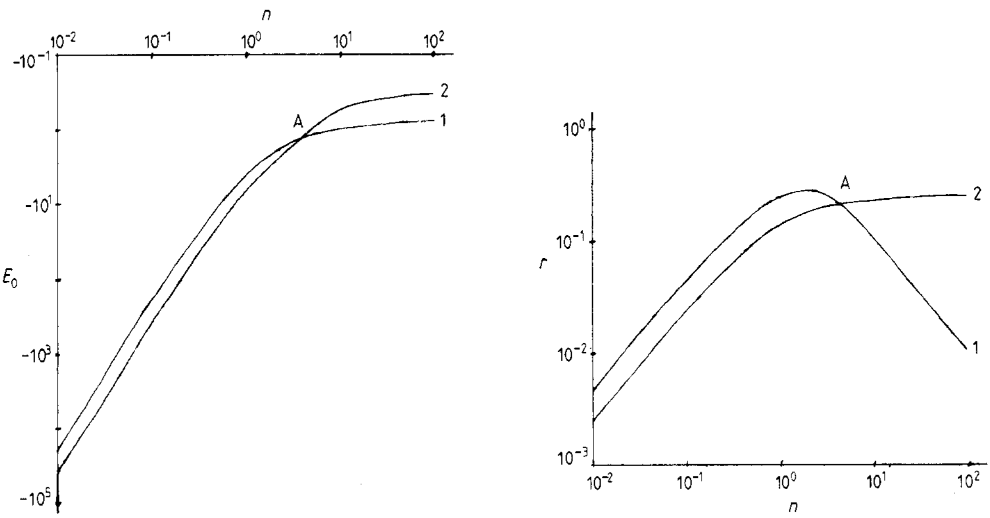

Figure 1. Curve (1): (+) branch, i.e. F_+^2, ν_+, y_-^2; curve (2): (−) branch, i.e. F_-^2, ν_-, y_+^2 (see the sign before the root of (12)). The point A lies in the interval of coordinates $4 < n < 5$; $-1.27 < E_0 < -1.15$. The shape of the curves does not vary appreciably with Λ/m^2, so we plot the curves only for $\Lambda/m^2 = 0$.

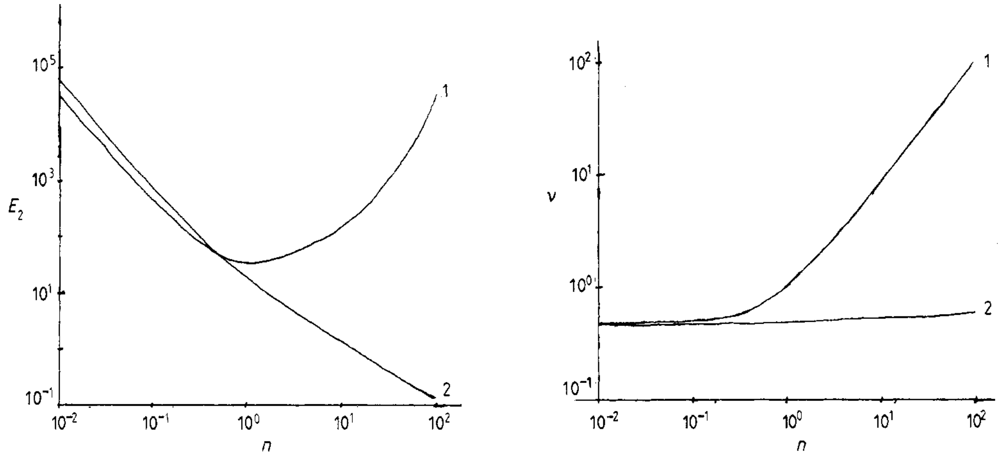

Figure 2. Curves (1) and (2) as in figure 1.

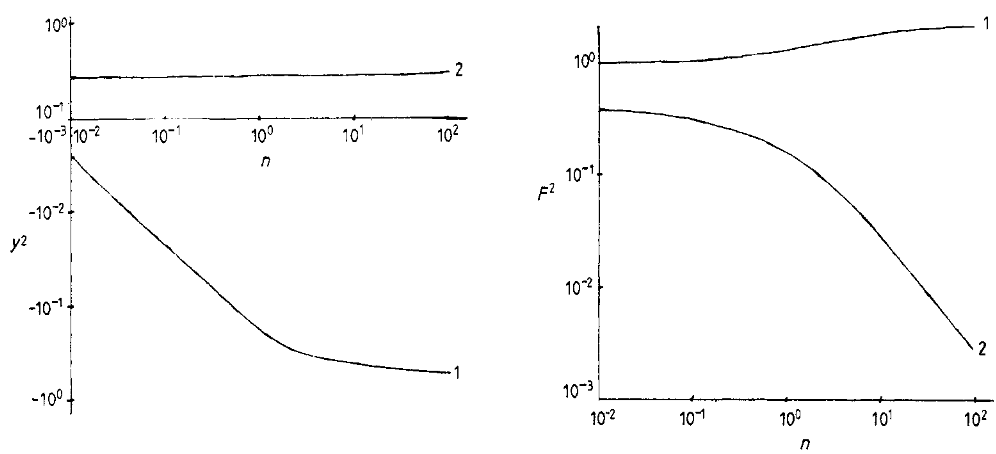

Figure 3. Curves (1) and (2) as in figure 1.

In the general case, the number of branches is equal to the number of solutions of (16). The calculations showed that the curves for $\Lambda/m^2 = \pm 1$ are only slightly different from the curves of the case $\Lambda/m^2 = 0$.

We see that different branches exist for all Λ/m^2. We postulate: a branch with the lowest energy density, for fixed Λ/m^2, near the initial state corresponds to a physical reality. Thus, to define the law of the scale factor evolution, it is necessary to choose at first the lowest energy state at the graph of 'energy' $E_0(n, \Lambda/m^2)$ near $\eta = 0$. This energy state is at the (+) or (−) branch (see (11)–(14)). By the chosen branch one finds the values of the functions r, y^2, F^2, ν, E_2, corresponding to the initial lowest energy state, at the rest of the graphs. The computation showed that the state with a minimal scale factor (for given, fixed n and Λ/m^2), i.e. the state of maximal compression, corresponds to the state with the lowest energy. However, the acceleration of the universe's expansion from this state would be less than if the universe had expanded from a higher energy state. If we chose n and Λ/m^2 such that the initial energy state is at the point of intersection of the (+) and (−) branches, then it is natural to believe that the state with a greater amplitude F^2 is realised.

Note that our calculations are valid at time $\eta < \eta_b$, where

$$\eta_b = \min((\nu(n, \Lambda/m^2))^{-1/2}, (|y^{-2}(n, \Lambda/m^2)|)^{1/2}).$$

It is easy to see from physical considerations that the computation is valid until the scale factor variation is small compared with the scale factor of the initial state. Moreover, the scale factor of an initial state depends on the parameters n and Λ/m^2; hence the characteristic time of the scale factor variation η_b must also depend on these parameters. When putting the energy $\varepsilon_s(\eta)$ equal to zero, we may correctly find the symmetry restoration moment, provided that

$$\eta_r^2(n, \Lambda/m^2) = -E_0/E_2 \ll \eta_b^2.$$

If $\eta_r^2 < \eta_b^2$ then one may state that the value η_r is found correct to the order of magnitude. From figures 1, 2, 3 we can see that for any n and Λ/m^2 it is possible to find η_r to the order of magnitude. When $n \geqslant 5$ the symmetry restoration moment is found reliably. Then $\eta_r^2/\eta_b^2 \sim 0.1$.

Now we present some examples of our computation. Suppose that masses are fixed and $|\Lambda| \approx 10^{-56}\ \text{cm}^{-2}$. Then our calculations are correct for the following values of λ:

$$\begin{aligned} &\text{Planck mass} \quad m = 10^{33}\ \text{cm}^{-1}: && 10^{-2} < \lambda < 10^{2}, \\ &\text{baryon mass} \quad m = 10^{13}\ \text{cm}^{-1}: && 10^{-42} < \lambda < 10^{-38}, \\ &\text{graviton mass} \quad m = 10^{-28}\ \text{cm}^{-1}: && 10^{-114} < \lambda < 10^{-110}. \end{aligned} \tag{22}$$

For example, let us take the field with baryon mass and $\lambda = 10^{-42}$. Then $\Lambda/m^2 \approx 10^{-80} \approx 0$ and $n = 100$. For these n and Λ/m^2 the lowest 'energy' level of an initial state is at the (+) branch: $E_0 \approx -1$. By the given (+) branch we find at the (+) branch of the graph $r(n)$: $m^2 a_m^2 = 0.011$; of the graphs y^2 and $\nu(n)$: $\nu = 100$, $\eta_b^2 = 0.01$; of the diagram $E_2(n)$: $E_2 \approx 3 \times 10^4$. Using these values we obtain

$$a_m = 0.1 \times 10^{-13}\ \text{cm}, \qquad \nu = 100, \qquad \eta_r^2 = 10^{-4} \times 0.33 \ll \eta_b^2. \tag{23}$$

So, the scale factor evolution law is

$$a = 0.1 \times 10^{-13}(1 + 100\eta^2)\ \text{cm}. \tag{24a}$$

Using figure 3 we have the law of the vacuum expectation value evolution

$$f^2 = 1.98(1 - 0.88\eta^2). \tag{24b}$$

The symmetry restoration will take place when the scale factor changes by

$$\Delta a = a_m \nu \eta_r^2 = 3.3 \times 10^{-17}\ \text{cm}. \tag{25}$$

The treatment of the massless scalar field in the universe may be carried out analytically. Solutions of the system (6) (formulae only for f and $\varepsilon_s(\eta)$ were given by Grib and Mostepanenko (1979)) are

$$f^2 = 1, \tag{26a}$$

$$\varepsilon_s(\eta) = -\tfrac{3}{2}\lambda a^4, \tag{26b}$$

$$\Lambda>0: \ a=(3/2\Lambda)^{1/2}[(1+4\Lambda h^2/3)^{1/2}\cosh[(4\Lambda/3)^{1/2}t]-1]^{1/2} \qquad (27a)$$

$$\Lambda<0: \ a=(3/2|\Lambda|)^{1/2}[1-(1-4|\Lambda|h^2/3)^{1/2}\cos[(4|\Lambda|/3)^{1/2}t]]^{1/2}, \qquad (27b)$$

where $h^2=-ka^4\varepsilon_s(\eta)/3$.

These formulae (24) and (27) define the non-singular cosmology. As is seen from (27b), when $\Lambda<0$, the cosmology with oscillating scale factor will take place. Maximorum $\max a\equiv a_{\rm mam}$ is easily found. A qualitative analysis of equation (6b) gives $a_{\rm mam}=(3/|\Lambda|)^{1/2}$. The maximum of a will be $\sim(3/|\Lambda|)^{1/2}$ too. In the limit $\Lambda\to 0$, (27) transforms to $a=(h^2+t^2)^{1/2}$.

Note that in the massless case formulae (27) describe the evolution of the model at all times; as for the massless case, the energy ($\varepsilon=-\frac{3}{2}\lambda a^4$) at all times of the model evolution remains negative. So, it is easy to check that (27) satisfy present observations. For instance, the Hubble constant (Weinberg 1972) at small times (i.e. when $(4|\Lambda|/3)^{1/2}t\ll 1$) is $H\sim t^{-1}$.

Let us carry out a qualitative analysis of (6a), when (18) is valid. It gives us that the trivial solution of (6a), $f=0$, is unstable. The existence of stable non-zero vacuum expectation values (24b) and (26a), to which the lowest energy levels $\varepsilon=3m^4E_0/\lambda$ and (26b) correspond, means that a spontaneous breakdown of a symmetry may be realised in the physical system. It is easy to make the following statements about the absence or presence of a spontaneous breakdown of symmetries.

(A) An arbitrary symmetry is spontaneously broken if its eigenvalue is negative.

(B) An arbitrary symmetry is conserved if its eigenvalue is positive.

We should like to make some remarks about the point (B). There are non-zero vacuum expectation values in our scheme in so far as we consider an open model of the universe. In other words: the negative integral of the energy (7) takes place in an open universe, because the vacuum energy must be zero in the pseudo-Euclidean universe when adding a constant (it follows from experiments on observations of small, so pseudo-Euclidean, domains of the universe). It is necessary to add the same constant to the vacuum energy while regarding an open type or a close universe. We do it in order to normalise to a pseudo-Euclidean universe. For a massless field the vacuum energy is zero for a pseudo-Euclidean and a close universe, as in these cases the point $f=\dot{f}=0$ is the stable point on the phase plane of equation (6a).

However, in spite of the existence of non-zero vacuum expectation values, i.e. the negative vacuum energy in an open-type universe, by reason of the point (B), the scalar field may exist in it with positive quantum numbers of all symmetries. (A broken symmetry may be arbitrary: *C, P, T, CP, G, CPT* and so on, depending on the concrete field. Several symmetries may be spontaneously broken simultaneously too.) This situation possibly means that there are tachyons in the theory. The physical sense of tachyons is not clear at present. So, we think that the discovery of those relic scalar particles $m\sim 10^{-65}$ g (Melnikov and Orlov 1979), when all the symmetries are conserved, can point out that the universe is not open.

Acknowledgments

The author is most grateful to Dr V N Melnikov for his interest in the work and highly useful discussions. I thank N A Sokolov and T S Pan'kiv for their help when doing numerical calculations.

References

Bogolubov N N and Shirkov D V 1976 *Introduction to the Quantum Fields Theory* (Moscow: Nauka) in Russian

Bronnikov K A, Melnikov V N and Stanukovich K P 1968 *Preprint ITP of AS of Ukrainian SSR* No 68–69

Callan C, Coleman S and Jackiw 1970 *Ann. Phys., NY* **59** 42

Chernikov N A and Tagirov E A 1968 *Ann. Inst. H. Poincaré* **9A** 109

Grib A A and Mostepanenko V M 1979 in *Problems of Gravitational and Elementary Particles Theory* vol 10 (Moscow: Atomizdat) in Russian p 74

Lee T D 1974 *Phys. Rep.* **9C** 145

Melnikov V N and Orlov S V 1979 *Phys. Lett.* **70A** 263

Stanukovich K P 1965 *Gravitational Field and Elementary Particles* (Moscow: Nauka) in Russian

Weinberg S 1972 *Gravitation and Cosmology* (New York: Wiley)